Kuwait University | Math 102 | June 18, 1997
Dept of Math & Comp Sc: | Final Examination | Duration: 2 hours

Answer all of the following questions.

1. (5 points) Let $f(x) = e^{2x} + 4e^{x} + 1$, $-\infty < x < \infty$. Show that f is one-to-one. State the domain and range of f^{-1} and compute $f^{-1}(x)$.

2. (4 points) If $f(x) = \ln(e^{x} + 1) + \tan^{-1} x$, $-\infty < x < \infty$, then find the equation of the tangent line to the graph of f^{-1} at the point $P(\ln 2, 0)$

3. (4 points) Find $\dfrac{dy}{dx}$ if

$$y = \left(\frac{(\cosh x)^{x} \sqrt{1 + \pi^{2x}}}{(\sec^{-1} x) \ln |x|} \right)^{\sqrt{2}}$$

4. (5 points) Evaluate $\lim\limits_{x \to \infty} \left[\frac{1}{x} + \frac{2}{\pi} \tan^{-1} x \right]^{x}$

5. Evaluate the following integrals(5 points each)

(a) $\displaystyle\int \frac{dx}{(x+1)^3 \sqrt{x^2 + 2x - 3}}$

(b) $\displaystyle\int x^3 \sqrt{x \sinh(\ln x)}\, dx$

(c) $\displaystyle\int_1^{\infty} \frac{\tan^{-1} x}{x^2}\, dx$

6. (5 points) If the curve C is given parametrically as

$$\left.\begin{aligned} x(t) &= \ln(1 - t) \\ y(t) &= 2 \sin^{-1} \sqrt{t} \end{aligned}\right\}, \quad \frac{1}{9} \le t \le \frac{1}{4}$$

then find the length of C.

7. (4 points) Identify and sketch the curve whose polar equation is $r = \dfrac{6}{2 - \cos\theta}$ by transforming to rectangular coordinates.

8. (4 points) Find the area inside the cardioid $r = 2(1 + \sin\theta)$ and outside the circle $r = 2\sin\theta$.

9. (4 points) Find a parametric equation of the line through the point $P(5, 0, -2)$ that is parallel to the planes $x - 4y + 2z = 0$ and $2x + 3y - z + 1 = 0$.